An Efficient PMU-based Fault Location Technique for Multi-Terminal Transmission Lines

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Abstract—This paper presents a new fault location technique for multi-terminal transmission lines using phasor measurement unit (PMU). A two-stage fault location model is proposed, along with defining nodal current unbalance, a fault location index. The first stage is the fault line selector stage, which uses the non-zero elements of nodal current unbalance to determine the fault line. The second stage is used to identify the exact fault distance. The computational burden of the proposed technique is very low because it provides an analytical solution and avoids iterative computations. The performance of this technique is thoroughly evaluated under various fault conditions. Very promising simulation results verify the accuracy and robustness of the proposed technique for multi-terminal transmission lines.

Index Terms—Fault location, multi-terminal transmission line, nodal current unbalance, phasor measurement unit.

I. INTRODUCTION

Accurate fault location on a transmission line can expedite repair of the faulted components, speed-up restoration, reduce outage time, and thus improve power system reliability [1]. Till now, the most common methods are one-terminal method and two-terminal method. One-terminal methods only use one-terminal voltage and current phasors, so the accuracies of fault location are normally adversely affected by the fault resistance and remote-terminal system impedance [2-5]. To improve the accuracy of fault location, two-terminal algorithms are developed [6-9].

With the development of modern power system, the transmission network is becoming more and more complicated. Three terminal and multi-terminal transmission lines inevitably appear and the existing one-terminal or two-terminal fault location algorithms are unable to determine which branch the fault occurs on. Several fault location methods for three terminal transmission lines have been proposed [10-13]. However, due to the complexity of the problem, only a few algorithms [14-18] for fault location in multi-terminal transmission lines have been proposed. Abe et al. [14] used a reactive power-based method to locate the exact fault position after the multi-terminal line was reduced into a two-terminal line containing the fault section. Funabashi et al. [15] used two different methods to locate the fault. However, results for three-phase and two-phase to ground faults weren’t reported. Sanderson et al. [16] can successfully identify the faulted section, but the exact fault location on the faulted section was not studied. Chi-Wen Liu et al [17] extended a two-terminal fault location technique to N-terminal transmission lines. It is suitable for any type of multi-terminal line. But, (N-1) two-terminal indices should be calculated, which increases the computational burden. Brahma [18] reported successful results by only using voltage measurements to locate fault, but the exact power source impedances must be available. The source impedances at line terminals are also required in [19-20].

For multi-terminal transmission lines, the main difficulty of fault location is to identify the faulted section. Once the faulted section is identified, the fault point can be located easily. These existing methods [14-18] may bring heavy computational burden or require source impedances to identify the faulted section. Therefore, a multi-terminal fault location technique can be promising when it is with low computational burden and avoids the use of source impedances. To achieve this objective, nodal current unbalance is defined firstly, and then used as a fault location index in this paper. Based on this index, a novel fault section selector is proposed to locate the fault on a multi-terminal transmission line. Simulation studies verify that the proposed technique is accurate and efficient under different fault conditions.

II. THE PROPOSED FAULT LOCATION INDEX

A. Generating the fault location index

Considering n–terminal transmission line depicted in Fig. 1, all nodes are classified into two types: terminal node $p$ \( (p = 1, 3, 5, ..., n−3, n−1, n) \), and tap node $q$ \( (q = 2, 4, 6, ..., n−4, n−2) \). We assume that every terminal node is equipped with a PMU; this assumption is common in the literature [14-18]. Therefore, the synchronized voltage and current phasors at all terminals are available.

Because the positive sequence is the only network sequence for all types of faults, the positive sequence measurements are utilized in this paper. All the quantities, if not specifically labeled, refer to positive sequence quantities. It is assumed that the transmission lines to be considered are not specifically labeled, refer to positive sequence quantities.
The effect of shunt capacitances is taken into account by admittance matrix. The admittance matrix for the system can be written as:

\[ Y_{new} = \begin{bmatrix} Y_{11} & \cdots & Y_{1n} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ Y_{n1} & \cdots & Y_{nn} & Y'_{n(n+1)} \\ 0 & \cdots & 0 & Y'_{(n+1)(n+1)} \end{bmatrix} \]  

(2)

The elements in \( Y_{new} \) related to node \( i \) and node \( j \) are different from those in the original matrix \( Y_{non} \). Equations (3)–(8) describe this difference:

\[ Y'_{ij} = Y_{ij} \cdot \frac{Y_{ij}}{Z_{ij}} + \frac{xY_{ij}}{2Z_{ij}} + \frac{1}{xZ_{ij}} \]  

(3)

\[ Y'_{ji} = Y_{ji} = 0 \]  

(4)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(5)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(6)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(7)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(8)

The elements in \( Y_{new} \) related to node \( i \) and node \( j \) are different from those in the original matrix \( Y_{non} \). Equations (3)–(8) describe this difference:

\[ Y'_i = Y_i \cdot \frac{Y_i}{Z_{ii}} + \frac{xY_i}{2Z_{ii}} + \frac{1}{xZ_{ii}} \]  

(3)

\[ Y'_j = Y_j = 0 \]  

(4)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(5)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(6)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(7)

\[ Y'_{(i+1)(i+1)} = Y_{(i+1)(i+1)} = -\frac{1}{xZ_{ij}} \]  

(8)

Except the above elements, the other elements in \( Y_{non} \) and \( Y_{new} \) are the same.

Hence, when a fault occurs on the line \( i-j \), the following equation can be formed:

\[ Y_{new} \begin{bmatrix} U_{n+1} \\ U_f \end{bmatrix} = \begin{bmatrix} I_{n+1} \\ I_f \end{bmatrix} \]  

(9)

Where, \( U_f \), \( I_f \) are the node voltage and current injection at fault point \( F \) respectively; \( U_{n+1} \) is the post-fault node voltage vector; \( I_{n+1} \) is the post-fault node current vector.

For the \( i^{th} \) and \( j^{th} \) rows in (9), we have:

\[ Y_{ii}U_i + \cdots + Y_{ij}U_j + \cdots + Y_{in}U_n + Y'_{(i+1)}U_f = I_i \]  

(10)

\[ Y'_{ii}U_i + \cdots + Y'_{ij}U_j + \cdots + Y'_{in}U_n + Y'_{(i+1)}U_f = I_i + \Delta I \]  

(11)

Equation (10) and (11) can be modified as follows:

\[ Y_{ii}U_i + \cdots + Y_{ij}U_j + \cdots + Y_{in}U_n + Y'_{(i+1)}U_f = I_i + \Delta I \]  

(12)

\[ Y'_{ii}U_i + \cdots + Y'_{ij}U_j + \cdots + Y'_{in}U_n + Y'_{(i+1)}U_f = I_i + \Delta I \]  

(13)

For convenience, equation (12) and (13) are simplified as:

\[ Y_{ii}U_i + \cdots + Y_{ij}U_j + \cdots + Y_{in}U_n + Y'_{(i+1)}U_f = I_i + \Delta I \]  

(14)

\[ Y'_{ii}U_i + \cdots + Y'_{ij}U_j + \cdots + Y'_{in}U_n + Y'_{(i+1)}U_f = I_i + \Delta I \]  

(15)

Where,

\[ \Delta I = (Y_{ii} - Y'_{ii})U_i + (Y_{ij} - Y'_{ij})U_j + Y'_{(i+1)}U_f \]  

(16)

\[ \Delta I = (Y'_{ii} - Y_{ii})U_i + (Y'_{ij} - Y_{ij})U_j + Y_{(i+1)}U_f \]  

(17)

Except the \( i^{th} \), \( j^{th} \) and \((n+1)^{th}\) rows, the other rows in (9) can be written as follows:

\[ Y_{ii}U_i + \cdots + Y_{ij}U_j + \cdots + Y_{in}U_n + Y_{(n+1)}U_{n+1} = I_k \]  

(18)

Where, \( k = 1, 2, \ldots, i-1, i+1, \ldots, j-1, j+1, \ldots, n-1, n \).

From (14), (15) and (18), the following equation can be established:
\[
\begin{bmatrix}
Y_{i1} & \ldots & Y_{in} & \ldots & Y_{in} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
Y_{nj} & \ldots & Y_{nj} & \ldots & Y_{nj} \\
\end{bmatrix}
\begin{bmatrix}
U_i \\
\vdots \\
U_j \\
\vdots \\
U_n \\
\end{bmatrix}
= 
\begin{bmatrix}
I_i \\
\vdots \\
I_j \\
\vdots \\
I_n \\
\end{bmatrix}
\]

(19)

Equation (19) can be rewritten as:
\[
\begin{bmatrix}
U_{n+1} \\
\vdots \\
U_n \\
\end{bmatrix}
= 
\Delta U_{n+1} + \Delta U_n
\]

(20)

Where
\[
\Delta U_{n+1} = \begin{bmatrix} 0, \ldots, 0, \Delta I_i, 0, \Delta I_j, 0, \ldots, 0 \end{bmatrix}^T
\]

(21)

From (21), we can observe the following characteristic: when a fault occurs on the line section \(i-j\), \(\Delta U_{n+1}\) has only two non-zero elements \(\Delta I_i\) and \(\Delta I_j\). By checking the non-zero elements of \(\Delta U_{n+1}\), we can identify the faulted line section.

Hence, \(\Delta U_{n+1}\) is defined as nodal current unbalance in (21), which will provide a simple and efficient index to locate the faulted section in multi-terminal lines.

From (20), in order to obtain \(\Delta U_{n+1}\), \(U_{n+1}\) and \(I_{n+1}\) must be known. When a fault occurs on the system in Fig.1, the voltage \(U_p\) and injected current \(I_p\) at every terminal node \(p\) \((p = 1, 3, 5, \ldots, n-3, n-1, n)\) are measured by PMU, and the current injection \(I_q\) at tap node \(q\) \((q = 2, 4, 6, \ldots, n-4, n-2)\) is zero, so \(I_{n+1}\) and \(U_{n+1}\) \((p = 1, 3, 5, \ldots, n-3, n-1, n)\) are known, only the voltage \(U_q\) at the tap node \(q\) \((q = 2, 4, 6, \ldots, n-4, n-2)\) is unknown.

In order to obtain \(U_q\), we firstly assume that the fault only occurs on the main line section \((1-2, 2-4, 4-6, \text{etc.})\). From the geometry of the multi-terminal lines in Fig.1, the following equation can be derived for terminal node \(p\) \((p = 3, 5, \ldots, n-3, n-1)\):
\[
Y_{pp} U_p + Y_{p(p-1)} U_{p-1} = I_p
\]

(22)

Since \(U_{p-1}\) is the sole unknown variable in (22), it can be easily obtained as follows:
\[
U_{p-1} = (I_p - Y_{pp} U_p)/Y_{p(p-1)}
\]

(23)

Because \(I_{n+1}\) and \(U_p\) \((p = 1, 3, 5, \ldots, n-3, n-1, n)\) are known by PMU measurements, so we can compute the voltage \(U_{p-1}\) of all node by (23). Once \(U_{n+1}\) and \(I_{n+1}\) are obtained, then \(\Delta I_{n+1}\) can be calculated by using (24).
\[
\Delta I_{n+1} = Y_{n+1} U_{n+1} - I_{n+1}
\]

(24)

However, the fault may also occur on tapped line section (line 2-3, 4-5, 6-7, etc.) in this case, the assumption is not satisfied. So we will discuss its application in the following two cases: a fault occurs on the main line section, and a fault occurs on the tapped line section. For each case, \(\Delta I_{n+1}\) have different characteristics.

\section*{B. Fault on the main line section}

As shown in Fig.4, the fault occurs on the main line section (line 1-2, 2-4, 4-6, etc.) In this case, the assumption holds. So, the unknown voltages at all the tap nodes can be calculated by (23) correctly.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4}
\caption{a fault occurs on the main line section \(i-j\).}
\end{figure}

If a fault occurs on the main line section \(i-j\), obviously the computed fault location index \(\Delta I_{n+1}\) will only have two non-zero elements as shown in Eq. (21).

More specifically, in Fig.1, main line sections consist of line \(1-2\) and \(l-(l+2)\) \((l \in 2, 4, \ldots, n-2)\), so \(\Delta I_{n+1}\) can be outlined as follows:
\[
\Delta I_{n+1} = \begin{bmatrix} [\Delta I_i, \Delta I_j, 0, \ldots, 0]^T \end{bmatrix} \quad \text{fault on line} \ 1-2
\]
\[
\begin{bmatrix} [0, \ldots, 0, \Delta I_i, 0, \Delta I_{l+2}, 0, \ldots, 0]^T \end{bmatrix} \quad \text{fault on line} \ \ l-(l+2)
\]

(25)

\section*{C. Fault on the tapped line section}

If a fault occurs on the tapped line section (line 2-3, 4-5, 6-7, etc.) such as the tapped line \(i-j\) as shown in Fig.5, the assumption does not hold. The voltage \(U_i\) at tap node \(i\) is calculated by (23) and the fault current \(I_f\) is not considered. So the calculated \(U_i\) is not the true voltage \(U_i\) of tap node \(i\), i.e. \(U_i \neq U_i\). In this case, \(\Delta I_{n+1}\) has different characteristic.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5}
\caption{a fault occurs on the tapped line section \(i-j\).}
\end{figure}

In Fig.5, the calculated voltage \(U_i\) at tap node \(i\) is calculated by (23), i.e.:
\[
U_i = (I_j - Y_{ji} U_j)/Y_{ji}
\]

(26)

Due to the effect of fault current \(I_f\), the calculated voltage \(U_i\) does not equal to the true voltage \(U_i\) at tap node \(i\). In this case, we have:
\[
U_{n+1} = \begin{bmatrix} U_1 \ldots U_i \ldots U_j \ldots U_n \end{bmatrix}
\]

(27)

Therefore, \(\Delta I_{n+1}\) can also be obtained as follows:
\[
\Delta I_{n+1} = Y_{n+1} U_{n+1} - I_{n+1}
\]

Next, we will analyze all non-zero elements of \(\Delta I_{n+1}\) in this case.

For node \(i\), we have:
\[
\Delta I_i = Y_i U_i + \ldots + Y_{1} U_{1} + \ldots + Y_{i-1} U_{i-1} + \ldots + Y_n U_n - I_i
\]

(28)
III. IDENTIFYING FAULT SECTION

Based on the theoretical analysis shown in (25) and (38), we propose a fault section selector for multi-terminal lines, which is illustrated by Fig. 6.

There are three cases for the fault section selector. For every case, \( \Delta I_{\text{rel}} \) has different characteristic which can be used to identify the faulted line section:

Case 1: \( \Delta I_{\text{rel}} \) has two nonzero elements

For case 1, we can identify that a fault occurs on the main line section, referred to Eq.(25).

More specifically, if the two nonzero elements correspond to node \( i \) and \( j \), the fault line section \( i-j \) is identified undoubtedly. Actually, it should be line section 1–2 or \( l-(l+2) \) (\( l \in 2,4,...,n-2 \)) as shown in Fig.1.

Case 2: \( \Delta I_{\text{rel}} \) has three nonzero elements

For case 2, we can identify that a fault occurs on the tapped line section, referred to Eq.(38).

If the three nonzero elements correspond to nodes \( n_1, i \) and \( n_2 \) (\( n_1 < i < n_2 \)), the fault line section \( i-j \) is identified. Actually, it should be line section \( l-(l+1) \) (\( l \in 2,4,...,n-2 \)) as shown in Fig.1.

Case 3: every element of \( \Delta I_{\text{rel}} \) is zero

For case 3, we can identify that no fault occurs.

IV. EXACT FAULT LOCATION

Once the faulted section \( i-j \) is identified, the next step is to locate the exact fault point on the faulted section.

Case 1: A fault occurs on the main line section \( i-j \).

In this case, the calculated \( \Delta I_i \) and \( \Delta I_j \) are non-zero elements, which is given in (16) and (17). Based on (16) and (17), we can eliminate the unknown variable \( U_{ji} \): 

\[
[\Delta I_i - (Y_{ii} - Y_{ij})U_i + (Y_{ij} - Y_{jj})U_j]Y_{(i+1)j} = 0
\]

From (3)-(8), we know that \( Y_{ii} \), \( Y_{ij} \), \( Y_{(i+1)j} \) and \( Y_{(i+1)j} \) are functions of fault location variable \( x \). Substitute (3)-(8) into (39), we have the following quadratic equation with one unknown variable \( x \):

\[
ax^2 + bx + c = 0
\]

In equation (40), the only variable to be solved is the fault distance \( x \), and \( 0 \leq x \leq 1 \). The coefficient \( a \), \( b \) and \( c \) can be calculated as follow:

\[
\begin{align*}
\alpha &= k(U_i - U_j) \\
b &= -k(U_i - U_j) + Z_{l_i} (\Delta I_i + \Delta I_j) \\
c &= -\Delta I_j Z_{l_i} \\
k &= \frac{Y_{n_1} Z_{l_i}}{2}
\end{align*}
\]

Where, \( Y_{l_i} \) and \( Z_{l_i} \) are the parameters of \( \pi \) equivalent model of line \( i-j \). \( U_i \), \( U_j \) are the post-fault voltages at node \( i \) and \( j \), which can be calculated by equation (23).

By solving (40), we can obtain the exact fault point \( x \):
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (42) \]

Although the solution \( x \) can equal to two values in (42), only the solution satisfying \( 0 \leq x \leq 1 \) is accepted as the fault point.

**Case 2:** A fault occurs on the tapped line section \( i-j \).

In this case, the true voltage \( U_i \) of tap node \( i \) can be recalculated correctly by using the connected tap node \((i-2)\) or \((i+2)\). For example, we can use tap node \((i-2)\) to obtain \( U_i \):

\[ Y_{(i-2)(i-4)}U_{(i-4)} + Y_{(i-2)(i-3)}U_{(i-3)} + Y_{(i-2)(i-2)}U_{(i-2)} + Y_{(i-2)(i)}U_j = 0 \quad (43) \]

From (43), \( U_i \) can be recalculated as follows:

\[ U_i = \left[ Y_{(i-2)(i-4)}U_{(i-4)} + Y_{(i-2)(i-3)}U_{(i-3)} + Y_{(i-2)(i-2)}U_{(i-2)} \right] / Y_{(i-2)(i)} \quad (44) \]

Once \( U_i \) is recalculated correctly by (44), \( \Delta I_{\text{ref}} \) is also recalculated by (24). Then, \( \Delta I_{\text{ref}} \) will become to have two non-zero elements \( \Delta I_{\text{f}} \) and \( \Delta I_{\text{r}} \), which is a similar problem as case I. Then, the fault point \( x \) can also be obtained by (42).

**V. CASE STUDIES**

**A. Simulation system 1**

In order to evaluate the proposed fault location algorithm, Power Systems Computer Aided Design (PSCAD) [21] software has been utilized to generate transient waveforms for faults of different types, locations and fault resistances on a 500 kV, 50Hz six-terminal transmission line system, which is shown in Fig. 7. The related parameters of this system are given in Appendix I. The fault location error is defined as follows:

\[ \text{Error}(\%) = \left( \frac{\text{Estimated location} - \text{Actual location}}{\text{The length of faulted section}} \right) \times 100\% \quad (45) \]

To demonstrate the correctness of the developed fault line selector, some typical cases are discussed firstly. Fig.8 shows the calculated index \( \Delta I_{\text{ref}} \) for a A-phase to ground fault (AG-fault) on the main line 4-6. This fault occurs on the point which is 70% away from node 4 with fault resistance 50 \( \Omega \). It can be observed from Fig.8 that \( \Delta I_4 \) at node 4 and \( \Delta I_6 \) at node 6 are much larger than zero, while the other current unbalances almost equal to zero. Based on the principle of fault location shown in Fig.6, it can be identified that the fault occurs on the main line section 4-6. Furthermore, the exact fault location \( x \) can be obtained by (42): \( x_1=0.69999\) \( \approx \). \( x_2=234.89433 \). Because \( 0 \leq x \leq 1 \), so \( x=0.69999 \approx 69.999\% \) is the exact fault point. Therefore, the fault location error is 0.001\%, which is negligible.

![Fig.7. simulated six-terminal transmission lines](image)

**Table I. All Simulated Faults**

<table>
<thead>
<tr>
<th>Fault section</th>
<th>Fault information</th>
<th>Fault location</th>
<th>Resistance (( \Omega ))</th>
<th>Fault Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1-2</td>
<td>AG</td>
<td>5% from 1</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ABG</td>
<td>50% from 1</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>95% from 1</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td>Line 2-3</td>
<td>ACG</td>
<td>5% from 2</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>95% from 2</td>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>Line 2-4</td>
<td>CG</td>
<td>5% from 2</td>
<td>50</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>50% from 2</td>
<td>200</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>ACG</td>
<td>95% from 2</td>
<td>500</td>
<td>8</td>
</tr>
<tr>
<td>Line 4-5</td>
<td>ABC</td>
<td>50% from 4</td>
<td>200</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>AB</td>
<td>95% from 4</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>Line 4-6</td>
<td>BCG</td>
<td>5% from 4</td>
<td>100</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>50% from 4</td>
<td>200</td>
<td>12</td>
</tr>
<tr>
<td>Line 6-7</td>
<td>BC</td>
<td>5% from 6</td>
<td>500</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>AC</td>
<td>50% from 6</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>Line 6-8</td>
<td>BCG</td>
<td>95% from 6</td>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>AC</td>
<td>5% from 6</td>
<td>100</td>
<td>16</td>
</tr>
<tr>
<td>Line 6-9</td>
<td>BC</td>
<td>50% from 8</td>
<td>200</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>50% from 8</td>
<td>100</td>
<td>18</td>
</tr>
<tr>
<td>Line 8-9</td>
<td>ACG</td>
<td>95% from 8</td>
<td>50</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>50% from 8</td>
<td>200</td>
<td>20</td>
</tr>
<tr>
<td>Line 8-10</td>
<td>CG</td>
<td>5% from 8</td>
<td>1000</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>ACG</td>
<td>95% from 8</td>
<td>50</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>95% from 8</td>
<td>20</td>
<td>23</td>
</tr>
</tbody>
</table>

* ABG denotes A and B-phase to ground.
* AG denotes A-phase to ground.
* AB denotes two-phase short circuit.
* ABC denotes three-phase short circuit to ground.

Fig.9 (a) shows the calculated index \( \Delta I_{\text{ref}} \) for a A-phase to ground fault (AG-fault) on the tapped line section 6-7. The fault occurs on the point which is 20% away from node 6 with fault resistance 50 \( \Omega \). It can be observed from Fig.9 (a) that
the current unbalances \( \Delta I_a \) at node 4, \( \Delta I_b \) at node 6, and \( \Delta I_c \) at node 8 are much larger than zero. Based on the principle of fault location shown in Fig.6, it can be identified that the fault occurs on the tapped line section 6-7.

Once the tapped line section 6-7 is identified as faulted section, the correct voltage \( U_{ss} \) at node 6 can be calculated by (44). Then \( \Delta U_{ss} \) can be recalculated by (24), shown in Fig.9 (b). From Fig.9 (b), \( \Delta U_a \) and \( \Delta U_c \) are much larger than zero. Using \( \Delta U_a \), \( \Delta U_c \), \( U_a \) and \( U_c \), we can obtain the exact fault point \( x \) by solving (42): \( x_1 = 19.997\% \), \( x_2 = 241.64668 \). Obviously, \( x_2 = 19.997\% \) is the fault point. Therefore, the fault location error is 0.003\%, which is pretty small.

To validate the robustness of the proposed fault location technique, more faults are simulated. Table I gives all tested faults. These faults in Table I cover all possible line sections and different types of faults with various fault resistances. The maximum fault location error under various fault conditions is well below 0.1%.

The fault location results under different source impedances of node 1 are given in Table III. In Table III, a phase-A to ground fault occurs on the main line section 2-4, and the fault point is 50% away from node 2. As seen from Table III, the proposed fault location technique is independent of source impedances.

### Table II

**Simulation Results Under Different Fault Conditions**

<table>
<thead>
<tr>
<th>Fault Number</th>
<th>Fault Location index ( \Delta I_i ), (i = 1, 2, ..., 10)</th>
<th>Fault Section selector</th>
<th>Fault Point locator</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>5.00</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>50.00</td>
<td>0</td>
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<tr>
<td>3</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>95.00</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>5.00</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>49.99</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>95.00</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>50.00</td>
<td>0</td>
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<tr>
<td>8</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>94.99</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>4.99</td>
<td>0.01</td>
</tr>
<tr>
<td>11</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>95.00</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
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<td>0</td>
</tr>
<tr>
<td>14</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
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<td>0</td>
</tr>
<tr>
<td>16</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
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<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
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<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
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<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
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<tr>
<td>22</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
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</tr>
<tr>
<td>23</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>49.99</td>
<td>0.01</td>
</tr>
<tr>
<td>24</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>94.98</td>
<td>0.02</td>
</tr>
<tr>
<td>25</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>4.99</td>
<td>0.01</td>
</tr>
<tr>
<td>26</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>27</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Line 1-2</td>
<td>95.00</td>
<td>0</td>
</tr>
</tbody>
</table>

* \( \circ \) represents zero element.
* \( \bullet \) represents non-zero element.
* All 0 error means that the error is less than 0.01%.

The fault location results using the proposed technique are given in Table II.

As seen from Table II, the proposed fault location algorithm provides excellent performance for all considered faults and it isn’t affected by different fault types, fault positions, and fault resistances. The maximum fault location error under various fault conditions is well below 0.1%.

### Table III

**Simulation Results Under Different Fault Conditions**

<table>
<thead>
<tr>
<th>Fault location index ( \Delta I_i ), (i = 1, 2, ..., 10)</th>
<th>Source Impedances of Node 1</th>
<th>Fault Point locator</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>1 p.u.</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>2 p.u.</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>3 p.u.</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>5 p.u.</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>10 p.u.</td>
<td>50.00</td>
<td>0</td>
</tr>
</tbody>
</table>

* 1 p.u. means the original source impedance of node 1, as shown in Table IX
* All 0 error means that the error is less than 0.01%.

Commonly, when applying the method to practical case, the realistic parameters involved may not be of high accuracy. So we give the sensitivity of the proposed method to system parameters in Table IV and Table V. In Table IV, a phase-A to ground fault occurs on the main line section 2-4 with resistance 100 \( \Omega \), and the fault point is 75% away from node 2. In Table V, a phase-A to ground fault occurs on the tapped line section 4-5 with resistance 100 \( \Omega \), and the fault point is 75% away from node 4. From Table IV and Table V, we can see that the proposed method is affected in some way, but it still shows pretty good results.

### Table IV

**Computational Results Under Different Parameters**

<table>
<thead>
<tr>
<th>Influence factors</th>
<th>Fault Point Locator</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The parameters involved are totally accurate</td>
<td>75.00</td>
<td>0</td>
</tr>
<tr>
<td>Line parameters error on fault section 2-4 is 5%</td>
<td>77.49</td>
<td>2.49</td>
</tr>
<tr>
<td>Line parameters error on fault neighboring section 2-3 is 5%</td>
<td>75.24</td>
<td>0.24</td>
</tr>
<tr>
<td>Line parameters error on fault section 2-4 and on fault neighboring section 2-3 are both 5%</td>
<td>80.91</td>
<td>5.91</td>
</tr>
</tbody>
</table>

### Table V

**Computational Results Under Different Parameters**

<table>
<thead>
<tr>
<th>Influence factors</th>
<th>Fault Point Locator</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The parameters involved are totally accurate</td>
<td>74.98</td>
<td>0.02</td>
</tr>
<tr>
<td>Line parameters error on fault section 4-5 is 5%</td>
<td>74.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Line parameters error on fault neighboring section 2-4 is 5%</td>
<td>82.96</td>
<td>7.96</td>
</tr>
<tr>
<td>Line parameters error on fault section 4-5 and on fault neighboring section 2-4 are both 5%</td>
<td>81.65</td>
<td>6.65</td>
</tr>
</tbody>
</table>
B. Simulation system 2

**TABLE VI**

<table>
<thead>
<tr>
<th>Tap Section</th>
<th>System 1 (km)</th>
<th>System 2 (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-3</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>4-5</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>6-7</td>
<td>150</td>
<td>30</td>
</tr>
<tr>
<td>8-9</td>
<td>100</td>
<td>20</td>
</tr>
</tbody>
</table>

**TABLE VII**

<table>
<thead>
<tr>
<th>Fault location index $\Delta I_i$ ($i = 1, 2, ..., 10$)</th>
<th>Fault section selector</th>
<th>Fault point locator</th>
<th>Err (%)</th>
<th>Results from the original manuscript</th>
<th>Fault point locator</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>● ● o o o o o o o o o</td>
<td>1-2</td>
<td>5.00</td>
<td>0</td>
<td>5.00</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>● ● o o o o o o o o o</td>
<td>1-2</td>
<td>50.00</td>
<td>0</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>● o o o o o o o o o o</td>
<td>1-2</td>
<td>95.00</td>
<td>0</td>
<td>95.00</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>● ● o o o o o o o o o</td>
<td>2-3</td>
<td>5.01</td>
<td>0.01</td>
<td>5.00</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>● o o o o o o o o o o</td>
<td>2-3</td>
<td>50.01</td>
<td>0.01</td>
<td>49.99</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>● ● o o o o o o o o o</td>
<td>2-3</td>
<td>95.01</td>
<td>0.01</td>
<td>95.00</td>
<td>0.01</td>
</tr>
<tr>
<td>7</td>
<td>● o o o o o o o o o o</td>
<td>2-4</td>
<td>5.00</td>
<td>0</td>
<td>5.00</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>o o o o o o o o o o o</td>
<td>2-4</td>
<td>50.00</td>
<td>0</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>o o o o o o o o o o o</td>
<td>2-4</td>
<td>95.00</td>
<td>0</td>
<td>94.99</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>o o o o o o o o o o o</td>
<td>4-5</td>
<td>4.98</td>
<td>0.02</td>
<td>4.99</td>
<td>0.01</td>
</tr>
<tr>
<td>11</td>
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<td>4-5</td>
<td>50.00</td>
<td>0</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>o o o o o o o o o o o</td>
<td>4-5</td>
<td>95.00</td>
<td>0</td>
<td>95.00</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
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<td>4-6</td>
<td>5.00</td>
<td>0</td>
<td>5.00</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
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<td>4-6</td>
<td>50.00</td>
<td>0</td>
<td>50.00</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>95.00</td>
<td>0</td>
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<tr>
<td>16</td>
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<td>6-7</td>
<td>4.98</td>
<td>0.02</td>
<td>5.00</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
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<td>6-7</td>
<td>49.98</td>
<td>0.02</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
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<td>6-7</td>
<td>95.00</td>
<td>0</td>
<td>95.00</td>
<td>0</td>
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<tr>
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<td>5.00</td>
<td>0</td>
<td>5.00</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
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<td>6-8</td>
<td>50.00</td>
<td>0</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
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<td>6-8</td>
<td>95.00</td>
<td>0</td>
<td>95.00</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>o o o o o o o o o o o</td>
<td>8-9</td>
<td>4.99</td>
<td>0.01</td>
<td>5.00</td>
<td>0</td>
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<td>0.03</td>
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<td>0.01</td>
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<tr>
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<td>0.03</td>
<td>94.98</td>
<td>0.02</td>
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<tr>
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<td>4.99</td>
<td>0.01</td>
<td>4.99</td>
<td>0.01</td>
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<tr>
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<td>0</td>
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<tr>
<td>27</td>
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<td>8-10</td>
<td>95.00</td>
<td>0</td>
<td>95.00</td>
<td>0</td>
</tr>
</tbody>
</table>

* ○ represents zero element.
* ● represents non-zero element.
* All 0 error means that the error is less than 0.01%.

In realistic systems, many taps are necessarily short. To validate the proposed technique, a similar system with short taps is simulated. The simulation system 2 is exactly the same with simulation system 1, except for the taps length. The difference between their taps length is shown in Table VI.

The faults simulated are exactly the same as listed in Table I. The results are shown in Table VII. From the Table VII, the proposed technique shows very promising results for transmission lines with short taps, though the errors are a little higher than errors from transmission lines with long taps.

VI. CONCLUSIONS

This paper presents an accurate and efficient fault location technique for multi-terminal transmission lines. The nodal current unbalance $\Delta I_{\text{final}}$, fault location index, is firstly defined, then, the special features of $\Delta I_{\text{final}}$ for two different fault cases are derived by detailed theory analysis. The main conclusions of this paper are summarized as follows:

- When a fault occurs on the main line section $i-j$, $\Delta I_{\text{final}}$ has two non-zero elements.
- When a fault occurs on the tapped line section $i-j$, $\Delta I_{\text{final}}$ has three non-zero elements.
- Once the faulted section is identified, the exact fault point $x$ can be obtained directly by (42), the computational burden of proposed approach is very low.
- The case studies verify that the proposed approach is robust and enough accurate for all tested faults.

The distinctive features of the proposed fault location technique are concluded as follows:

- It is an improvement that the proposed fault location technique for multi-terminal lines avoids the use of source impedances in the formulation.

- The proposed technique performs with very high accuracy for all types of faults at different fault locations. Classification of fault types and selection of fault phase are not required.

- The proposed technique is practically immune to the fault resistance and is free of pre-fault conditions.

REFERENCES


APPENDIX I

The parameters of the six-terminal transmission lines are given in the following tables.

### TABLE VIII

<table>
<thead>
<tr>
<th>Section</th>
<th>Length (km)</th>
<th>( R_s ), ( X_s ), ( B_s ) (( \Omega / km ))</th>
<th>( R_o ), ( X_o ), ( B_o ) (( \Omega / km ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>100</td>
<td>( R_s = 3.5744 \times 10^{-2} )</td>
<td>( R_o = 3.6315 \times 10^{-2} )</td>
</tr>
<tr>
<td>2-4</td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-6</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-8</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8-9</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE IX

<table>
<thead>
<tr>
<th>Source</th>
<th>( Z_1 (\Omega) )</th>
<th>( Z_2 (\Omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 = 1 \angle 0^\circ )</td>
<td>0.238+j6.72</td>
<td>2.378+j10.1</td>
</tr>
<tr>
<td>( E_2 = 1 \angle 10^\circ )</td>
<td>0.155+j5.95</td>
<td>1.786+j7.58</td>
</tr>
<tr>
<td>( E_3 = 1 \angle 20^\circ )</td>
<td>0.367+j6.84</td>
<td>3.256+j10.58</td>
</tr>
<tr>
<td>( E_4 = 1 \angle 30^\circ )</td>
<td>0.132+j6.95</td>
<td>1.286+j8.58</td>
</tr>
<tr>
<td>( E_5 = 1 \angle 35^\circ )</td>
<td>0.338+j8.19</td>
<td>2.833+j11.12</td>
</tr>
</tbody>
</table>

### REFERENCES


### BIOGRAPHIES

**Quanyuan Jiang** (M’10) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Huazhong University of Science and Technology, Wuhan, China, in 1997, 2000, and 2003 respectively.

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